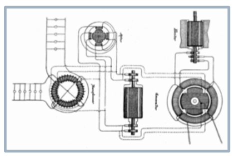
Watershed moment

Energy network will undergo similar architectural transformation that phone network went through in the last two decades to become the world's largest and most complex IoT



Tesla: multi-phase AC

1888

deregulation started

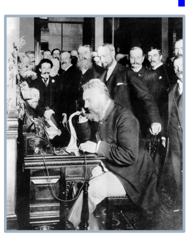
1980-90s

IoT

both started as natural monopolies
both provided a single commodity
both grew rapidly through two WWs

1876

Bell: telephone



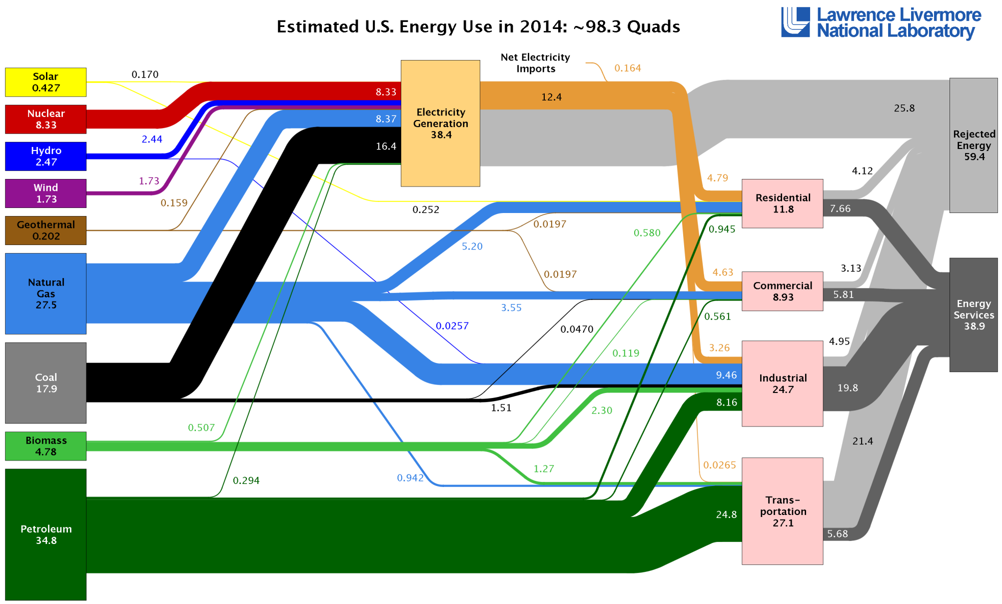
1980-90s

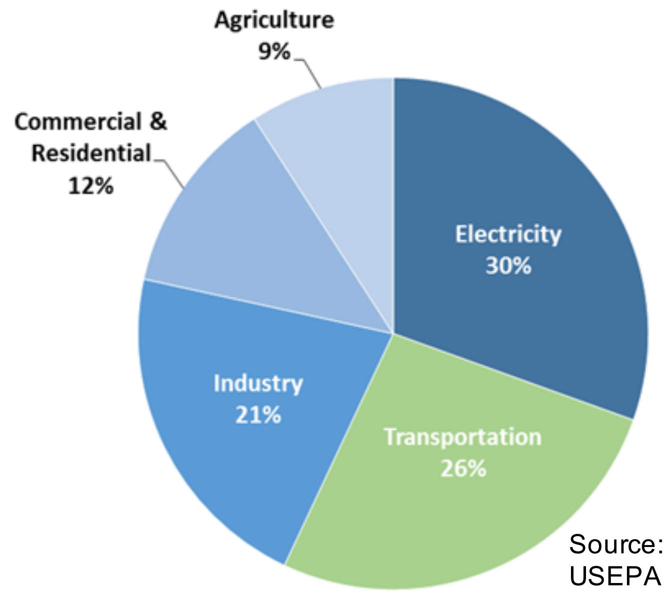
deregulation started

1969: DARPAnet

convergence to Internet

Electricity gen & transportation





They consume the most energy

- Consume 2/3 of all energy in US (2014)

They emit the most greenhouse gases

- Emit >1/2 of all greenhouse gases in US (2014)

To drastically reduce greenhouse gases

- Generate electricity from renewable sources
- Electrify transportation

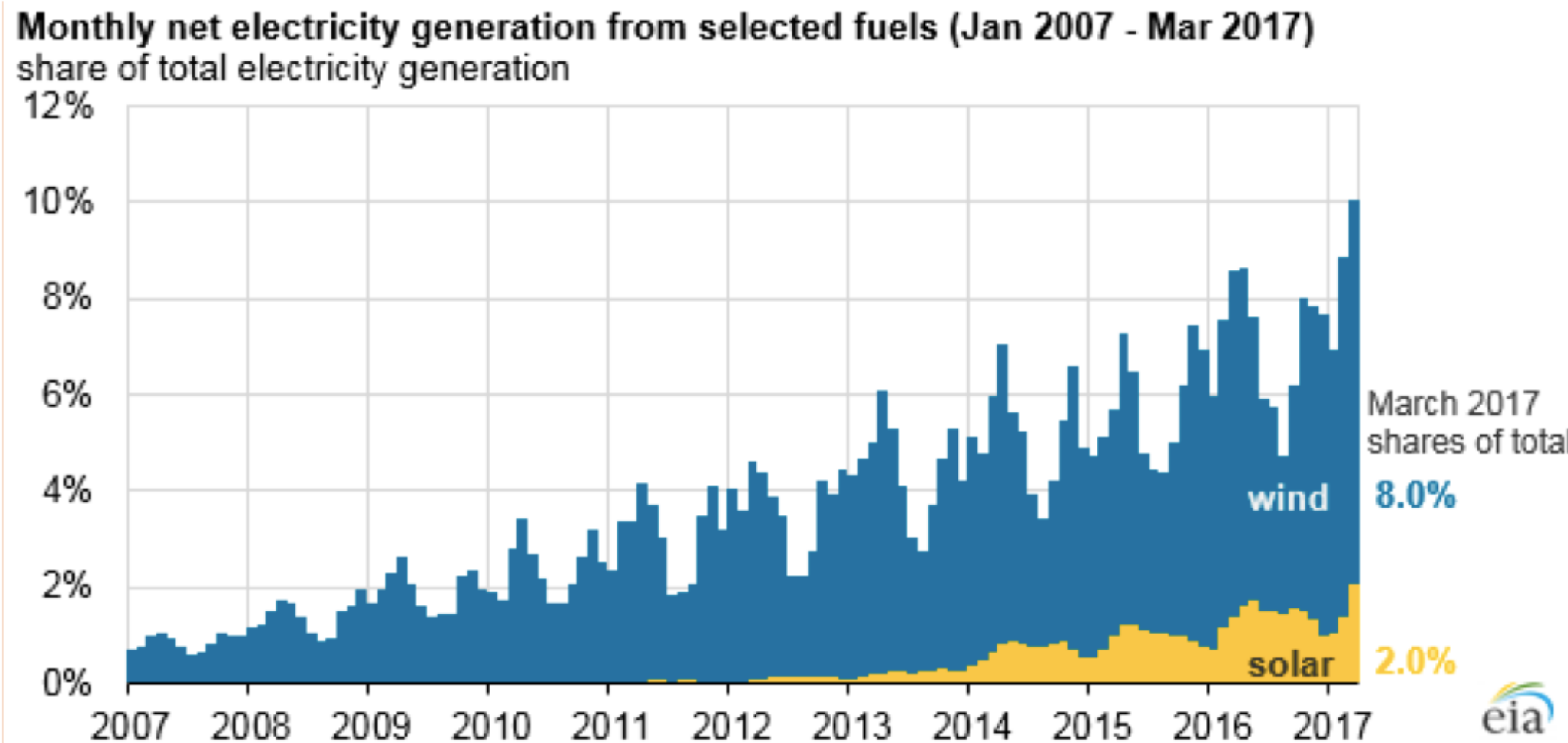


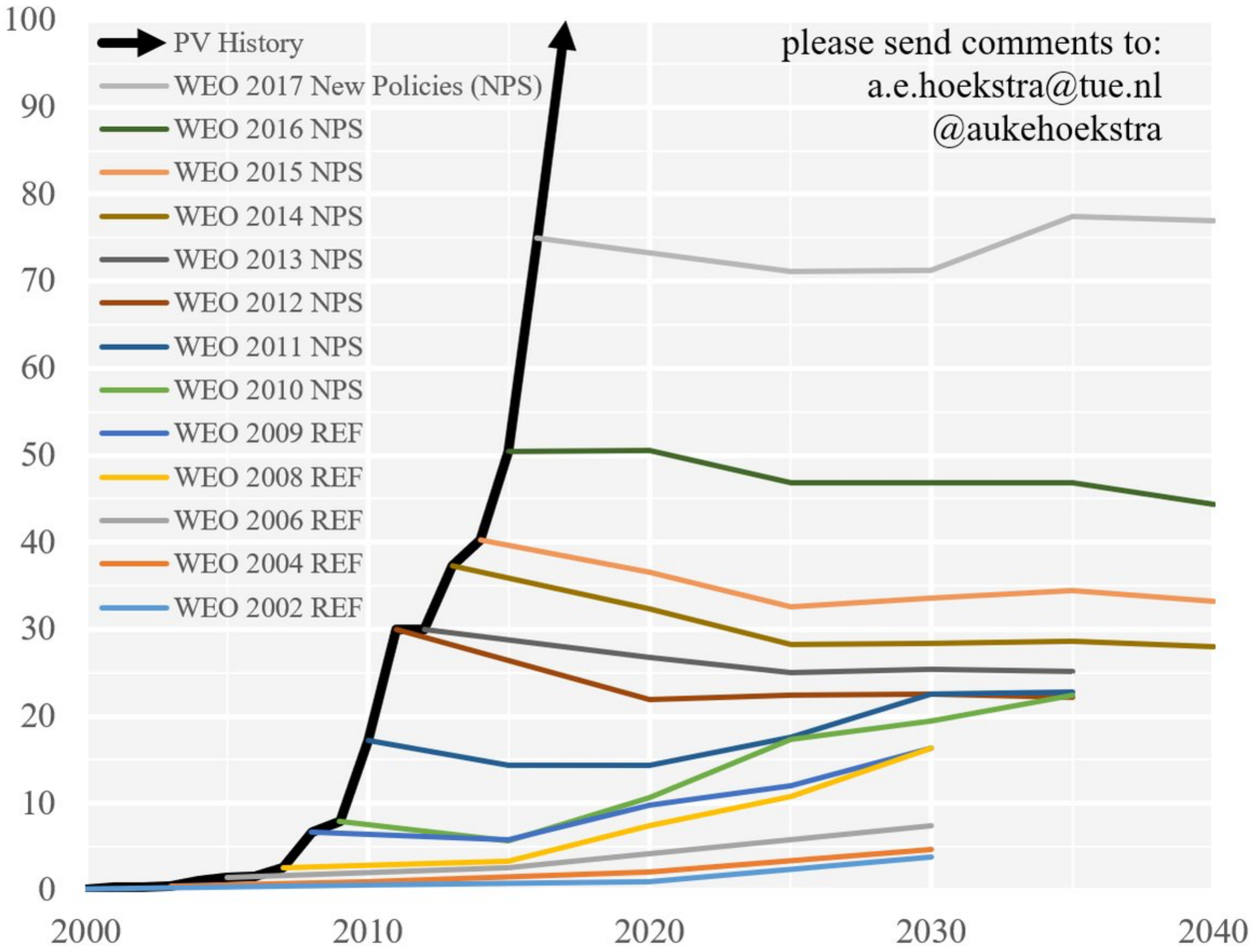
Figure 1: For the first time, in March 2017 solar supplied 2% of the U.S. electricity demand, while wind and solar combined accounted for 10% of the U.S. electricity generation. (Source: EIA)

DoE SETO 2030 cost target (unsubsidized cost in location with avg US solar resources):

- Utility-scale PV: 3c / kWh
- Commercial rooftop PV: 4c / kWh
- Residential rooftop PV: 5c / kWh
- Concentrating solar power w storage: 5c / kWh

Annual PV additions: historic data vs IEA WEO predictions

In GW of added capacity per year - source International Energy Agency - World Energy Outlook



Autonomous energy grid

Computational challenge

- nonlinear models, nonconvex optimization

Scalability challenge

- billions of intelligent DERs

Increased volatility

- in supply, demand, voltage, frequency

Limited sensing and control

- design of/constraint from cyber topology

Incomplete or unreliable data

- local state estimation & system identification

Data-driven modeling and control

- real-time at scale

many other important problems, inc. economic, regulatory, social, …

Ben Kroposki, 2007 https://www.nrel.gov/grid/autonomous-energy.html

Outline

Relaxations of AC OPF

- Dealing with nonconvexity

Realtime AC OPF

- Dealing with volatility

Optimal placement

- Dealing with limited sensing/control

Relaxations of AC OPF

dealing with nonconvexity


Bose (UIUC)


Chandy


Farivar (Google)


Gan (FB)


Lavaei (UCB)


Li (Harvard)

many others at & outside Caltech …

Low, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, ...

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Raphson, interior point, ...

$$\min \; c(x) \quad \text{s.t.} \quad F(x) = 0, \; x \le \overline{x}$$

Optimal power flow

$$
\begin{array}{lll}
\min & \operatorname{tr}\left(CVV^H\right) & \text{gen cost, power loss} \\
\text{over} & (V, s, l) & \\
\text{subject to} & s_j = \operatorname{tr}\left(Y_j^H VV^H\right) & \text{power flow equation} \\
& l_{jk} = \operatorname{tr}\left(B_{jk}^H VV^H\right) & \text{line flow} \\
& \underline{s}_j \le s_j \le \overline{s}_j & \text{injection limits} \\
& \underline{l}_{jk} \le l_{jk} \le \overline{l}_{jk} & \text{line limits} \\
& \underline{V}_j \le |V_j| \le \overline{V}_j & \text{voltage limits}
\end{array}
$$

- Y_j^H describes network topology and impedances
- s_j is net power injection (generation) at node j

Optimal power flow

min	$\mathrm{tr}\left(CVV^H\right)$	gen cost, power loss
over	(V, s, l)	
subject to	$s_j = \mathrm{tr}\left(Y_j^H VV^H\right)$	power flow equation
	$l_{jk} = \mathrm{tr}\left(B_{jk}^H VV^H\right)$	line flow
	$\underline{s}_j \leq s_j \leq \overline{s}_j$	injection limits
	$\underline{l}_{jk} \leq l_{jk} \leq \overline{l}_{jk}$	line limits
	$\underline{V}_j \leq \lvert V_j \rvert \leq \overline{V}_j$	voltage limits

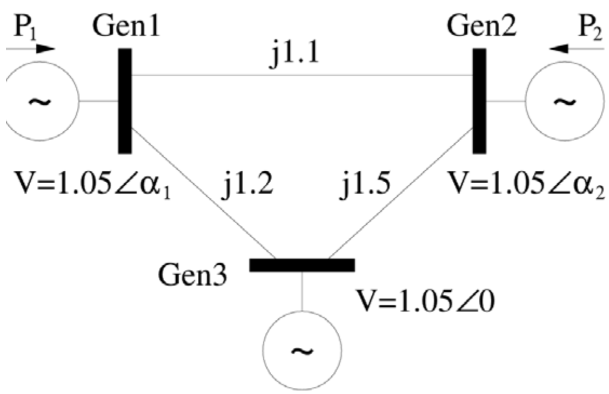
nonconvex feasible set (nonconvex QCQP)

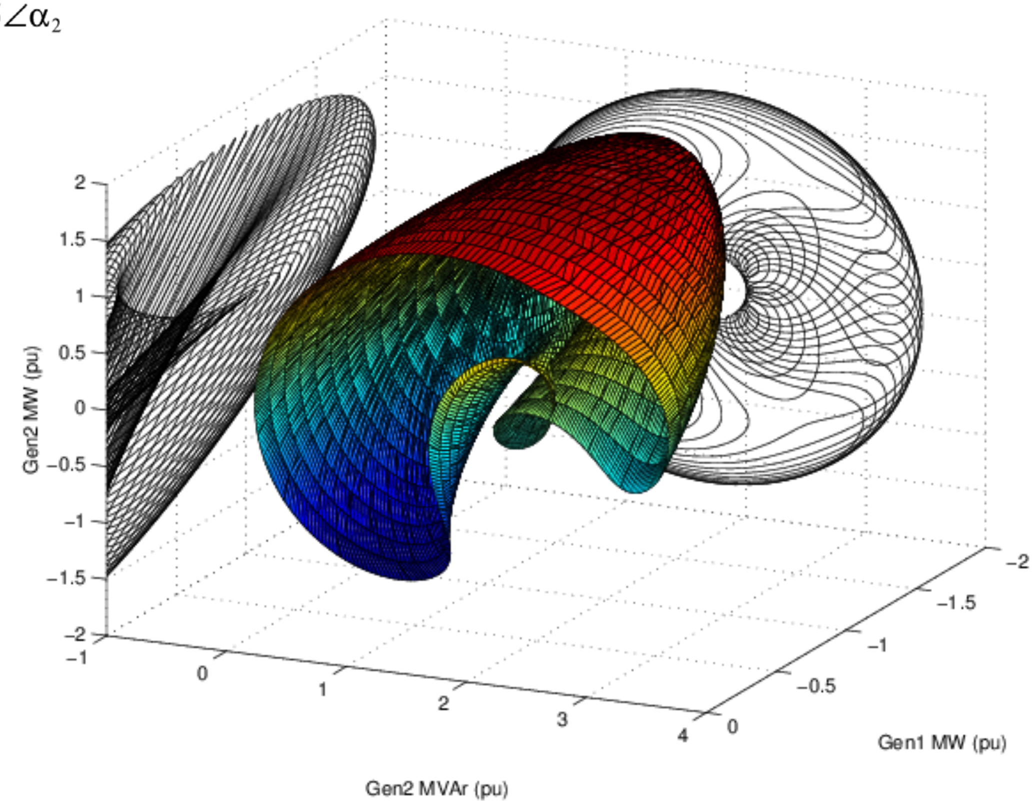
- Y_j^H not Hermitian (nor positive semidefinite)
- C is positive semidefinite (and Hermitian)

Optimal power flow

OPF problem underlies numerous applications

- nonlinearity of power flow equations ➔ nonconvexity





Ian Hiskens, Michigan

Dealing with nonconvexity

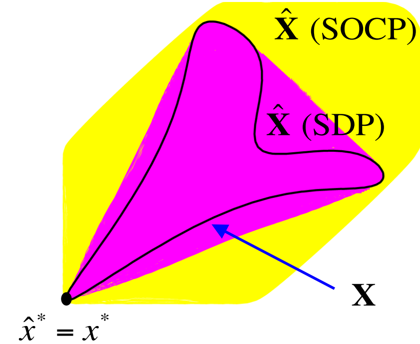
Linearization

- DC approximation

Convex relaxations

- Semidefinite relaxation (Lasserre hierarchy)
- QC relaxation (van Hentenryck)
- Strong SOCP (Sun)

Equivalent feasible sets



$$\min \quad \mathrm{tr}\, CVV^H$$

$$\text{subject to} \quad \underline{s}_j \le \mathrm{tr}\left(Y_j^H VV^H\right) \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j$$

quadratic in V
linear in W

Equivalent problem:

$$\min \quad \mathrm{tr}\, CW$$

$$\text{subject to} \quad \underline{s}_j \le \mathrm{tr}\left(Y_j^H W\right) \le \bar{s}_j \qquad \underline{v}_j \le W_{jj} \le \bar{v}_j$$

$$W \ge 0, \quad \mathrm{rank}\ W = 1$$

convex in W
except this constraint

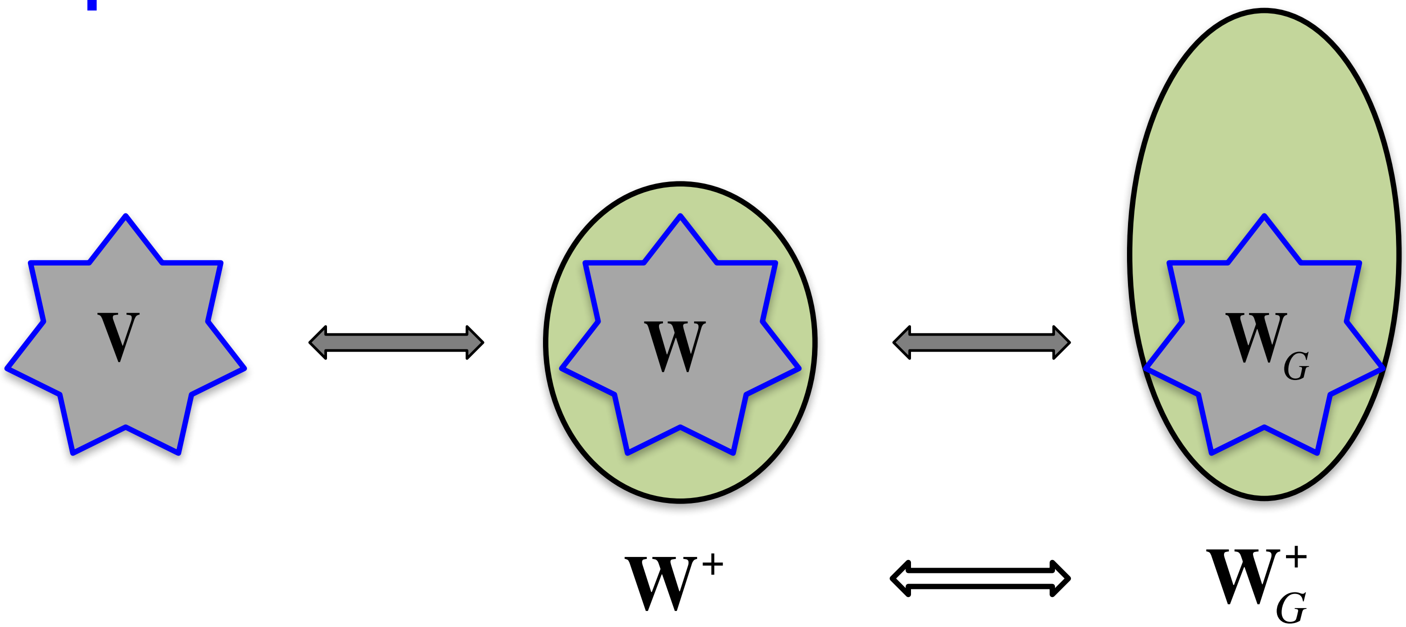
Solution strategy

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{\hat{x} \in \mathbf{X}^+} f(\hat{x})$$

If optimal solution $\hat{x}^*$ satisfies easily checkable conditions, then optimal solution x^* of OPF can be recovered

Equivalent relaxations



<u>Theorem</u>

- Radial G: SOCP is equivalent to SDP ($\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}_G^+$)
- Mesh G: SOCP is strictly coarser than SDP

For radial networks: always solve SOCP !

Exact relaxation

For radial networks, sufficient conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

Exact relaxation

QCQP (C, C_k)

$$\begin{aligned} &\text{min} && \operatorname{tr}\left(C x x^H\right) \\ &\text{over} && x \in \mathbf{C}^n \\ &\text{s.t.} && \operatorname{tr}\left(C_k x x^H\right) \le b_k \qquad k \in K \end{aligned}$$

graph of QCQP

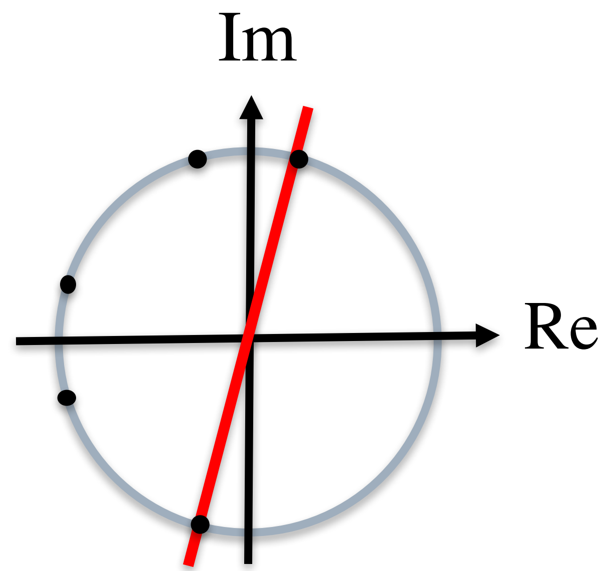
$G(C, C_k)$ has edge (i, j) $\Leftrightarrow$

$C_{ij} \neq 0$ or $\left[C_k\right]_{ij} \neq 0$ for some k

QCQP over tree

$G(C, C_k)$ is a tree

Exact relaxation



QCQP $\left(C, C_k\right)$

$$\begin{array}{lll} \min & \mathrm{tr}\left(Cxx^H\right) & \\ \text{over} & x \in \mathbf{C}^n & \\ \text{s.t.} & \mathrm{tr}\left(C_k xx^H\right) \leq b_k & k \in K \end{array}$$

Key condition

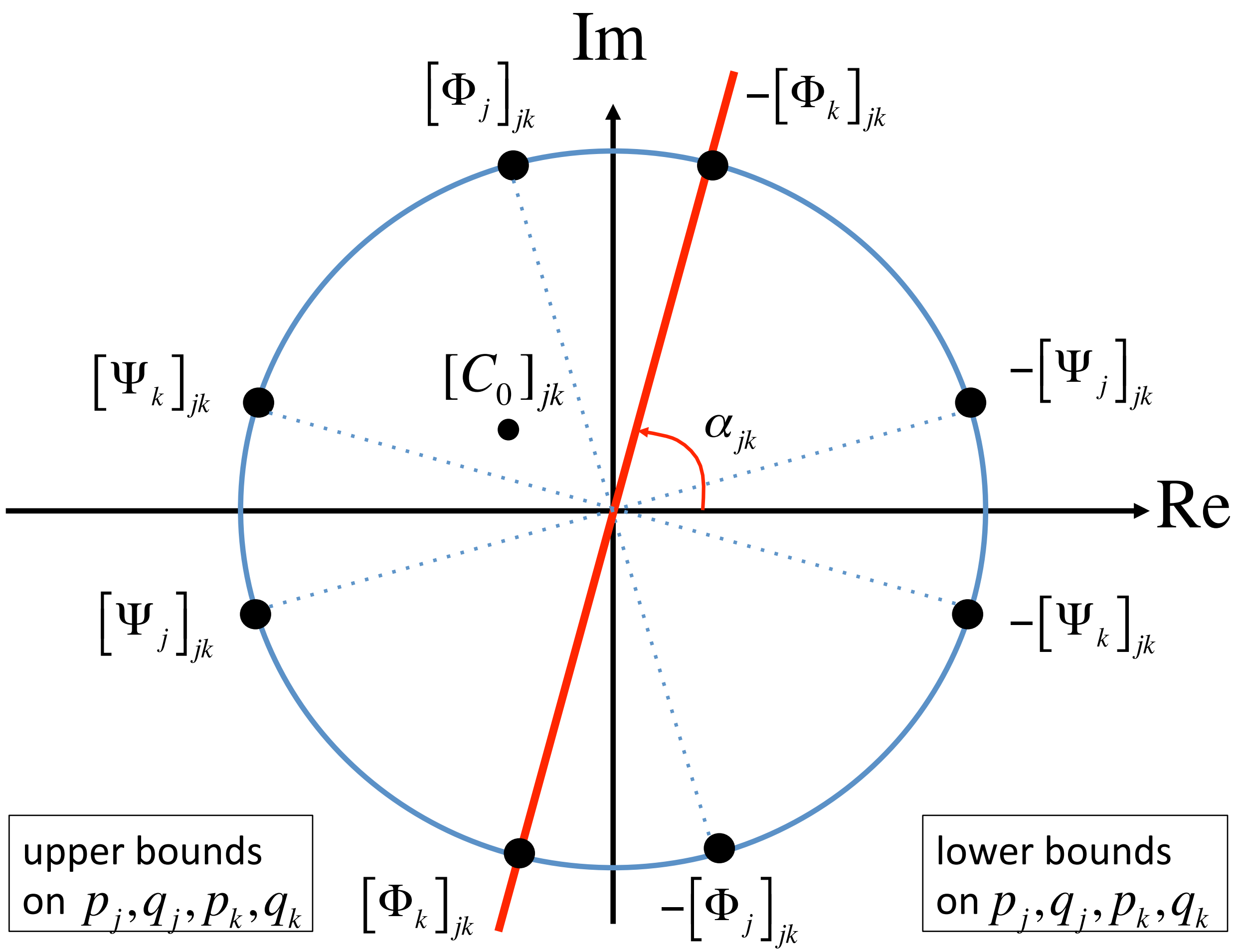
$$i \sim j: \quad \left(C_{ij}, [C_k]_{ij}, \ \forall k\right) \text{ lie on half-plane through } 0$$

Theorem

SOCP relaxation is exact for QCQP over tree

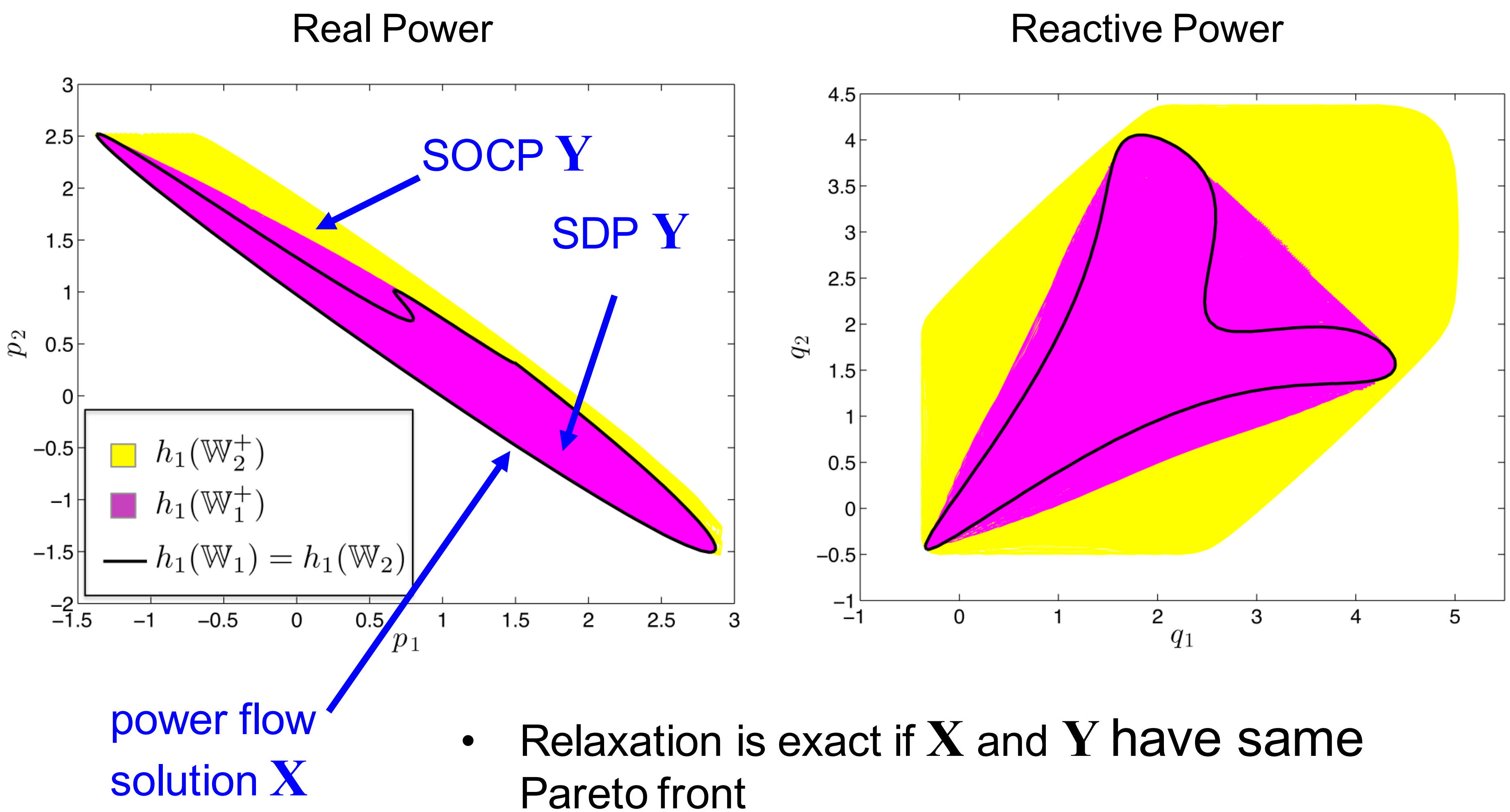
Bose et al 2012, 2014
Sojoudi, Lavaei 2013

Implication on OPF



Not both lower & upper bounds on real & reactive powers at both ends of a line can be finite

Example



- Relaxation is exact if $\mathbf{X}$ and $\mathbf{Y}$ have same Pareto front
- SOCP is faster but coarser than SDP

Bose, Low, Teeraratkul, Hassibi TAC 2015

Potential benefits

IEEE test systems		SDP cost	MATPOWER cost
Syst.	$\mathrm{rank}(\overline{X}_0)$	J°	$\overline{J}$
9	1	5296.7	5296.7
30	1	576.9	576.9
118	1	129661	129661
14A	1	**8092.8**	**9093.8**

12.4% lower cost than solution from nonlinear solver MATPOWER

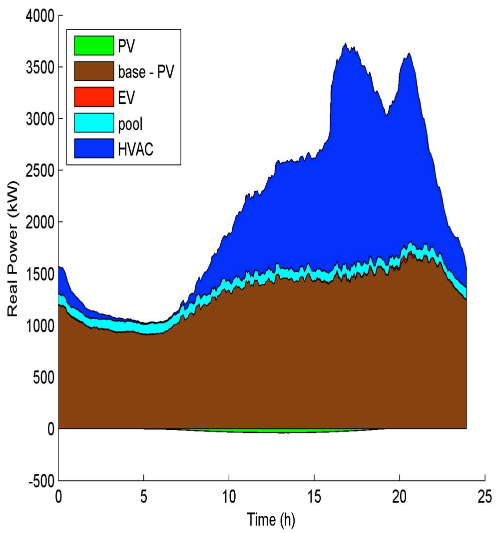
[Louca, Seiler, Bitar 2013]

Potential benefits

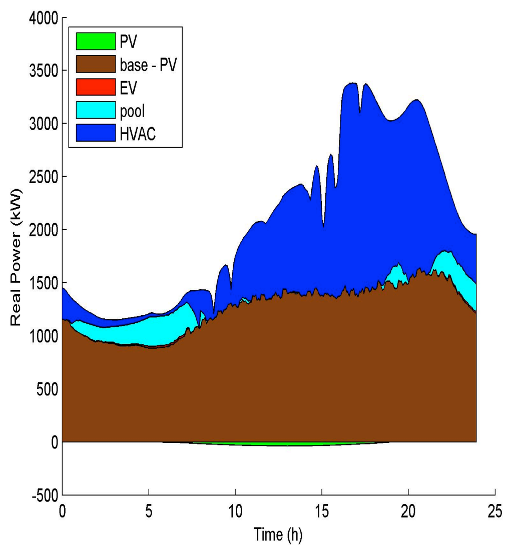
Case study on an SCE feeder

- Southern California
- 1,400 residential houses, ~200 commercial buildings
- Controllable loads: EV, pool pumps, HVAC, PV inverters
- Formulated as an OPF problem, multiphase unbalanced radial network

baseline



optimized



peak load reduction: 8%
energy cost reduction: 4%

Realtime AC OPF

for tracking

Gan (FB)

Tang (Caltech)

Dvijotham (DeepMind)

See also: Dall'Anese et al, Bernstein et al, Hug & Dorfler et al, Callaway et al

Gan & L, JSAC 2016
Tang et al, TSG 2017

Literature

Static OPF:

- Gan and Low, JSAC 2016
- Dall'Anese, Dhople and Giannakis, TPS 2016
- Arnold et al, TPS 2016
- A. Hauswirth, et al, Allerton 2016

Time-varying OPF:

- Dall'Anese and Simonetto, TSG 2016
- Wang et al, TPS 2016
- Tang, Dvijotham and Low, TSG 2017
- Tang and Low, CDC 2017

Earlier relevant work on voltage control

- Survey: Molzahn et al, TSG 2017

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x, \; y$$

$$\text{s. t.}$$

controllable devices (x)

uncontrollable state (y)

OPF

$$\begin{aligned} \min \quad & c_0(y) + c(x) \\ \text{over} \quad & x,\ y \\ \text{s.t.} \quad & F(x,y) = 0 \end{aligned}$$

power flow equations

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x,y) = 0$$

power flow equations

$$y \le \overline{y}$$

operational constraints

$$x \in X := \left\{ \underline{x} \le x \le \overline{x} \right\}$$

capacity limits

$$\text{Assume: } \frac{\partial F}{\partial y} \ne 0 \qquad \Rightarrow \qquad y(x) \quad \text{over } X$$

OPF

$$\min_{x} \quad c_0(y(x)) + c(x)$$

$$\text{s. t.} \quad y(x) \le \overline{y}$$

$$x \in X := \{\underline{x} \le x \le \overline{x}\}$$

Theorem [Huang, Wu, Wang, & Zhao. TPS 2016]
For DistFlow model, controllable (feasible) region

$$\{x \mid y(x) \le \overline{y}, x \in X\}$$

is convex (despite nonlinearity of $y(x)$)

Static OPF

$$\begin{array}{ll} \min & f(x, y(x);\ \mu) \\ \text{over} & x \in X \end{array}$$

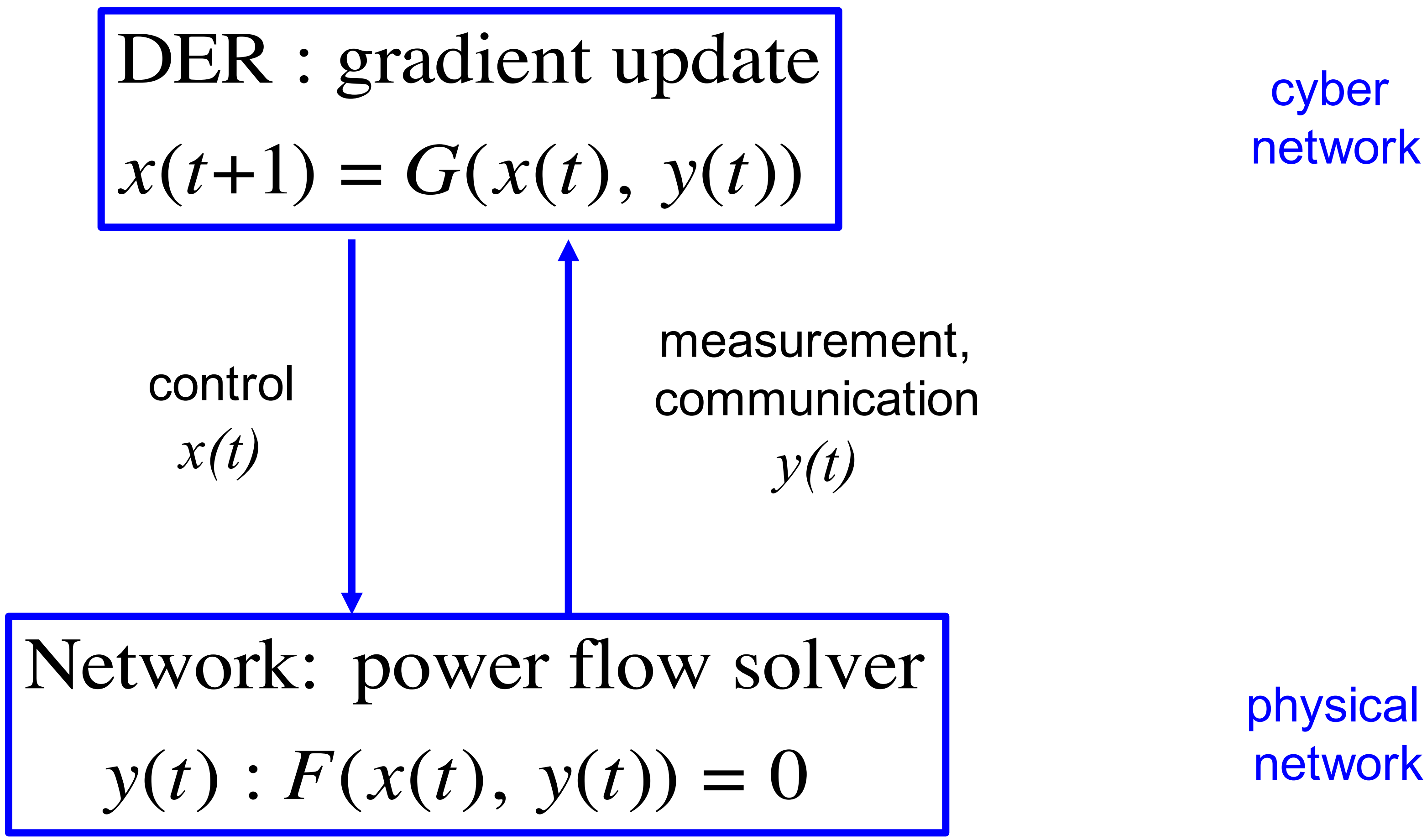
gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial f}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Gan & Low, JSAC 2016]

Online (feedback) perspective



- Explicitly exploits network as power flow solver
- Naturally tracks changing network conditions

Drifting OPF

$$\min_{x} \quad c_0(y(x)) + c(x)$$
$$\text{s.t.} \quad y(x) \le \overline{y}$$
$$x \in X$$

static OPF

$$\min_{x} \quad c_0(y(x), \gamma_t) + c(x, \gamma_t)$$
$$\text{s.t.} \quad y(x, \gamma_t) \le \overline{y}$$
$$x \in X$$

drifting OPF

Drifting OPF

$$\begin{array}{ll} \min & f_t(x, y(x);\ \mu_t) \\ \text{over} & x \in X_t \end{array}$$

Quasi-Newton algorithm:

$$x(t+1) = \left[x(t) - \eta\left(H(t)\right)^{-1} \frac{\partial f_t}{\partial x}(x(t)) \right]_{X_t} \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Tang, Dj & Low, 2017]

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

control error

Tracking performance

$$\text{error} \;:=\; \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \;\leq\; \frac{\varepsilon}{\sqrt{\lambda_M / \lambda_m} - \varepsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

avg rate of drifting

- of optimal solution
- of feasible injections

Tracking performance

$$\text{error} \;:=\; \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \;\le\; \frac{\varepsilon}{\sqrt{\lambda_M/\lambda_m} - \varepsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

error in Hessian approx

Tracking performance

$$\text{error} \;:=\; \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \;\leq\; \frac{\varepsilon}{\sqrt{\lambda_M / \lambda_m} - \varepsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

"condition number" of Hessian

Tracking performance

$$\text{error} \; := \; \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \; \le \; \frac{\varepsilon}{\sqrt{\lambda_M / \lambda_m} - \varepsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

"initial distance" from $x^{*}(t)$

Implementation

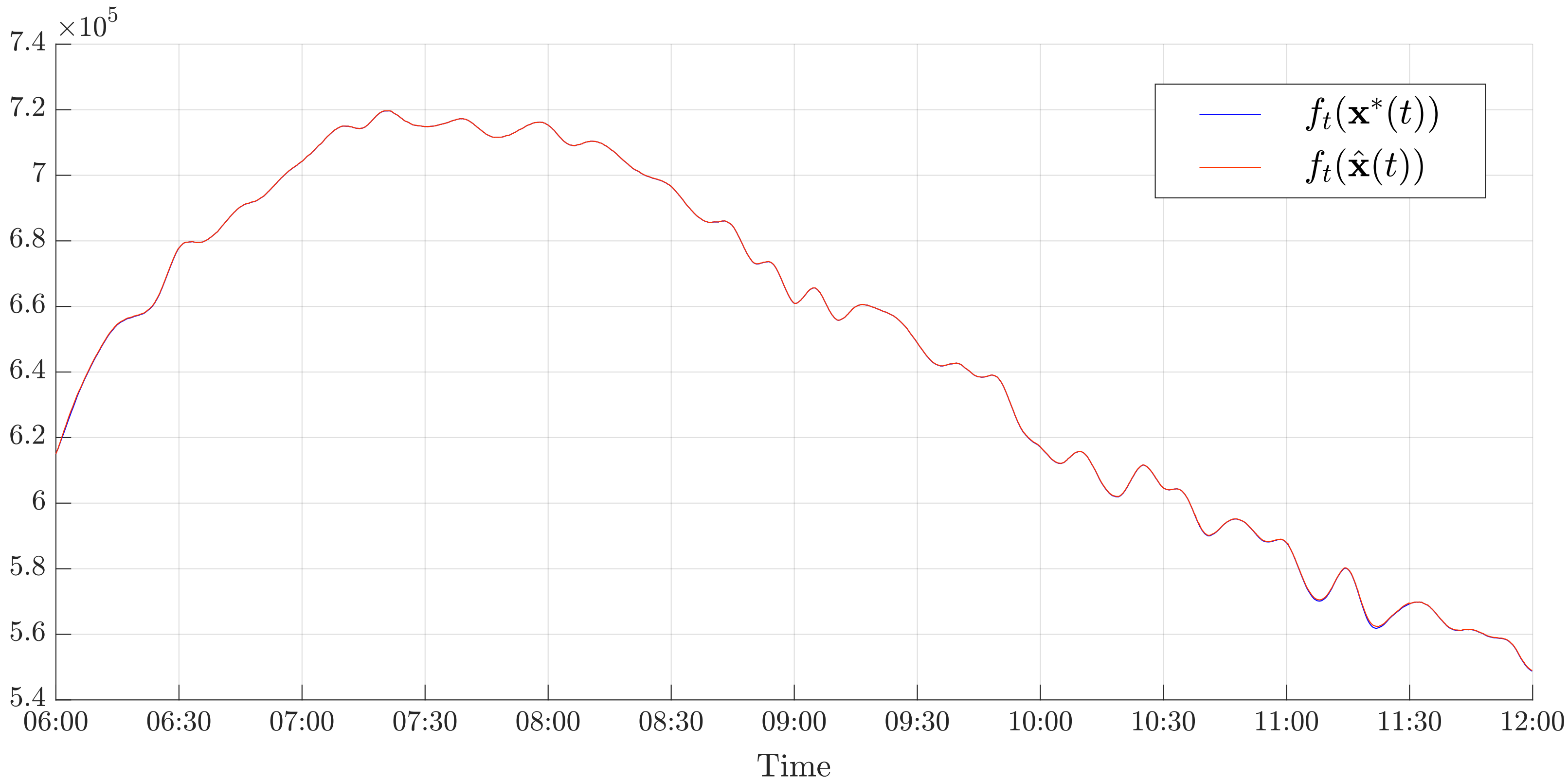
Implement L-BFGS-B

- More scalable
- Handles (box) constraints X

Simulations

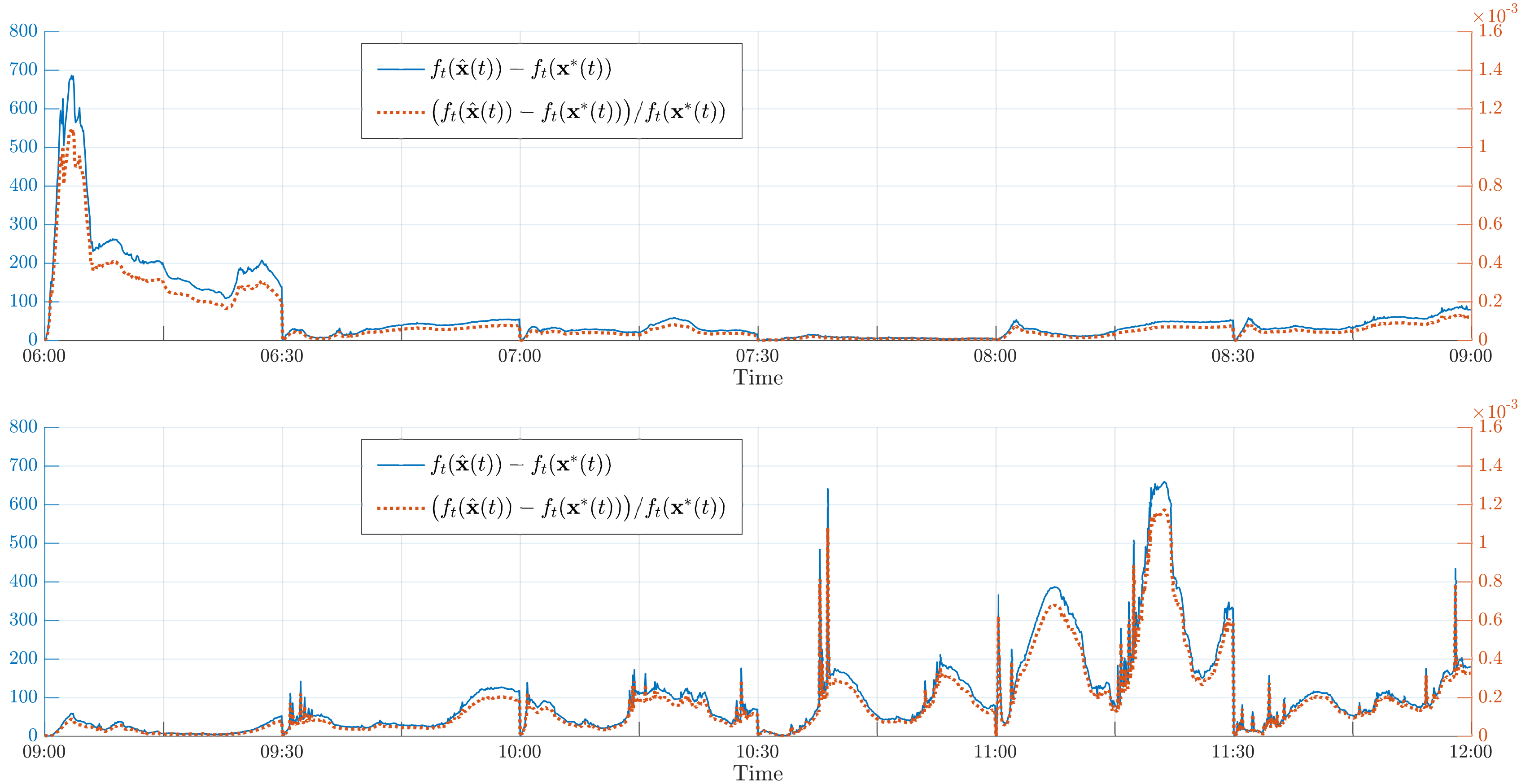
- IEEE 300 bus

Tracking performance



IEEE 300 bus

Tracking performance



IEEE 300 bus

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization [feedback control]

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Optimal placement
dealing with limited sensing/control

Guo (Caltech)

Guo & Low CDC 2017

Summary

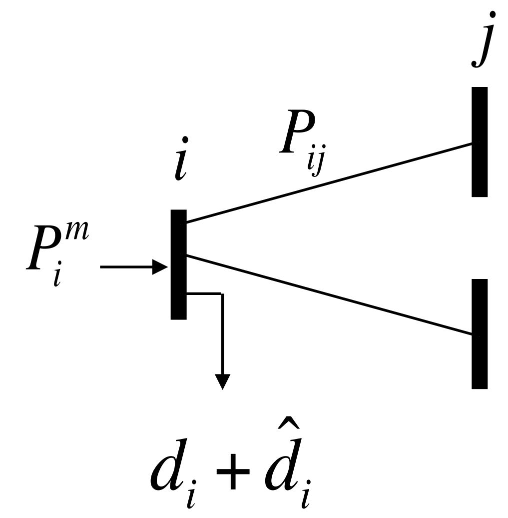
Characterization of controllability and observability

- of swing dynamics
- in terms spectrum of graph Laplacian matrix

Implications on optimal placement of controllable DERs and sensors

- set covering problem

Network model



swing dynamics:

$$-M_j \dot{\omega}_j = 1_{\mathcal{F}}(j)\hat{d}_j + 1_{\mathcal{U}}(j) d_j - P_j^m + \sum_{e \in \mathcal{E}} C_{je} P_e$$

$$\dot{P}_{ij} = B_{ij}(\omega_i - \omega_j)$$

controllable DER

$$y_j = 1_{\mathcal{S}}(j)\omega_j$$

frequency sensor

weighted Laplacian matrix

$$L = M^{-1/2} C B C^T M^{-1/2}$$

Algebraic coverage

spectral decomposition of L

$$L = Q\Lambda Q^T$$

eigenvectors of L

$$Q = \left[\beta_1 \ \cdots \ \beta_n\right]$$

algebraic coverage of bus j

$$\text{cov}(j) := \left\{ s \ \middle| \ \beta_{sj} \neq 0 \right\}$$

Controllability

Theorem

Swing dynamics is controllable if and only if

- L has a simple spectrum holds a.s.
- controllable DERs have full coverage

$$\bigcup_{j \in U} \mathrm{cov}(j) = \{\text{all buses}\}$$

Observability

Theorem

Swing dynamics is observable if and only if

- L has a simple spectrum holds a.s.
- frequency sensors have full coverage

$$\bigcup_{j \in S} \mathrm{cov}(j) = \{\text{all buses}\}$$

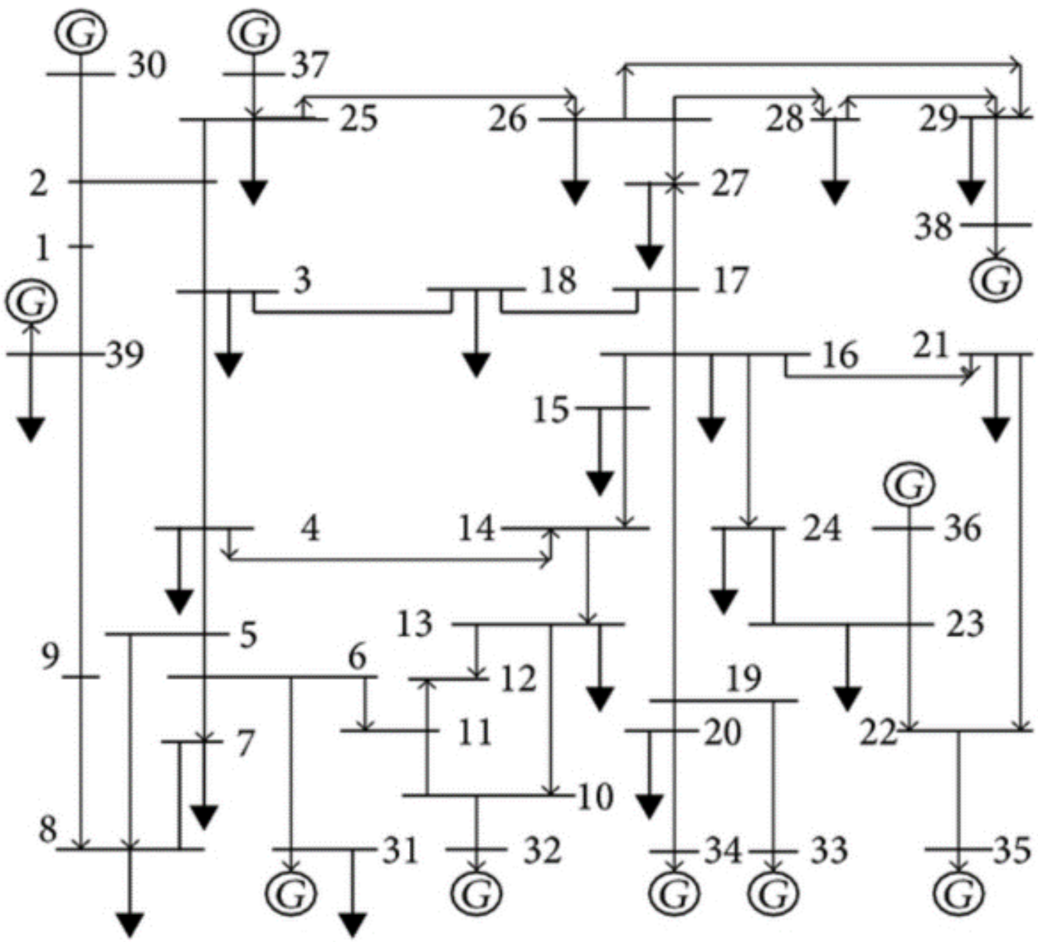
Application

Optimal placement of DER & frequency sensors

- set covering problem
- always install sensors at buses with controllable DERs, and vice versa

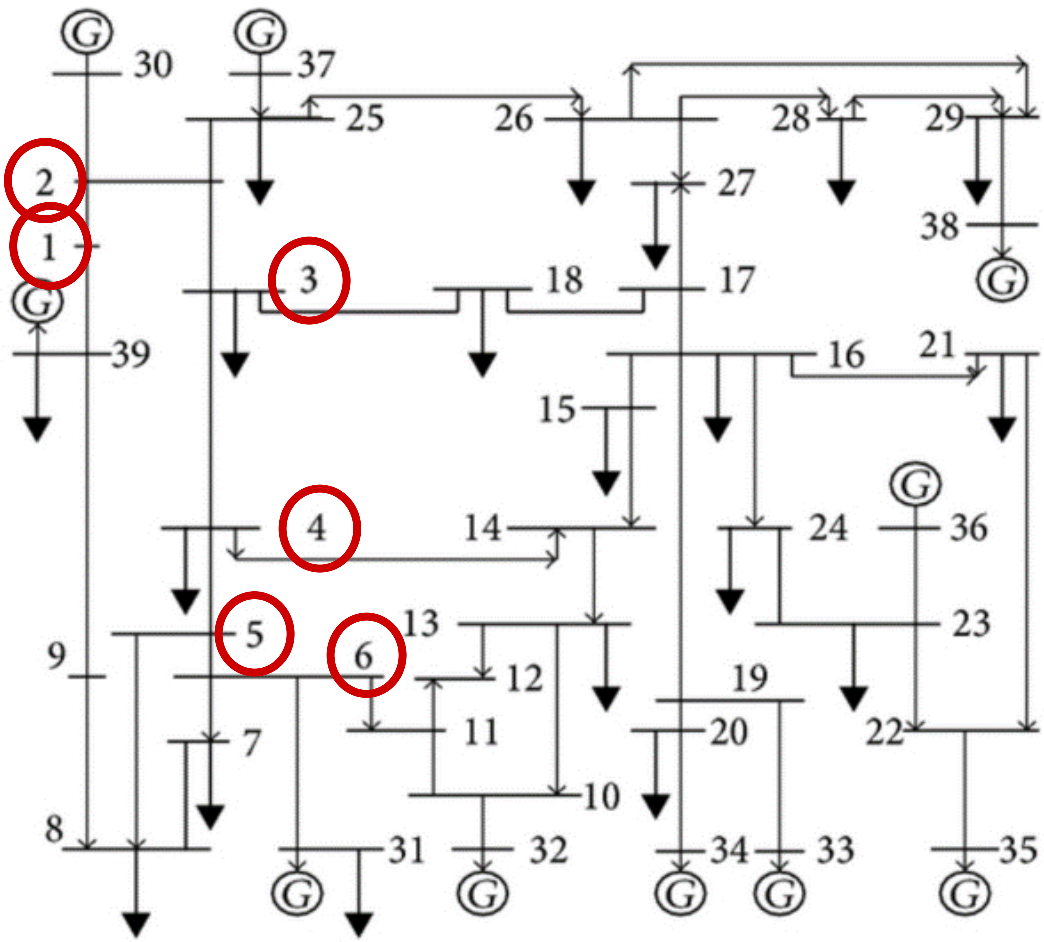
Example 2 – Control Coverage

- Which choice provides controllability?



Example 2 – Control Coverage

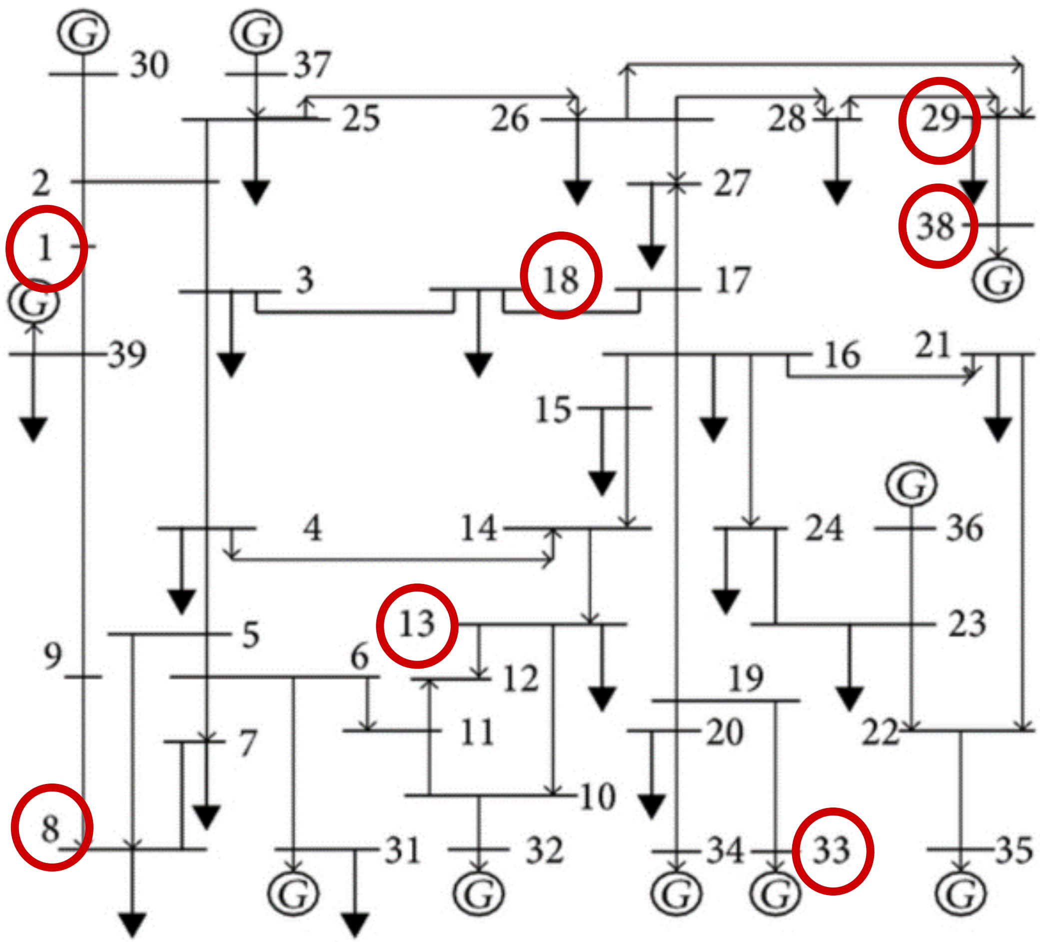
- Which choice provides controllability?



(a) {1,2,3,4,5,6}

Example 2 – Control Coverage

- Which choice provides controllability?

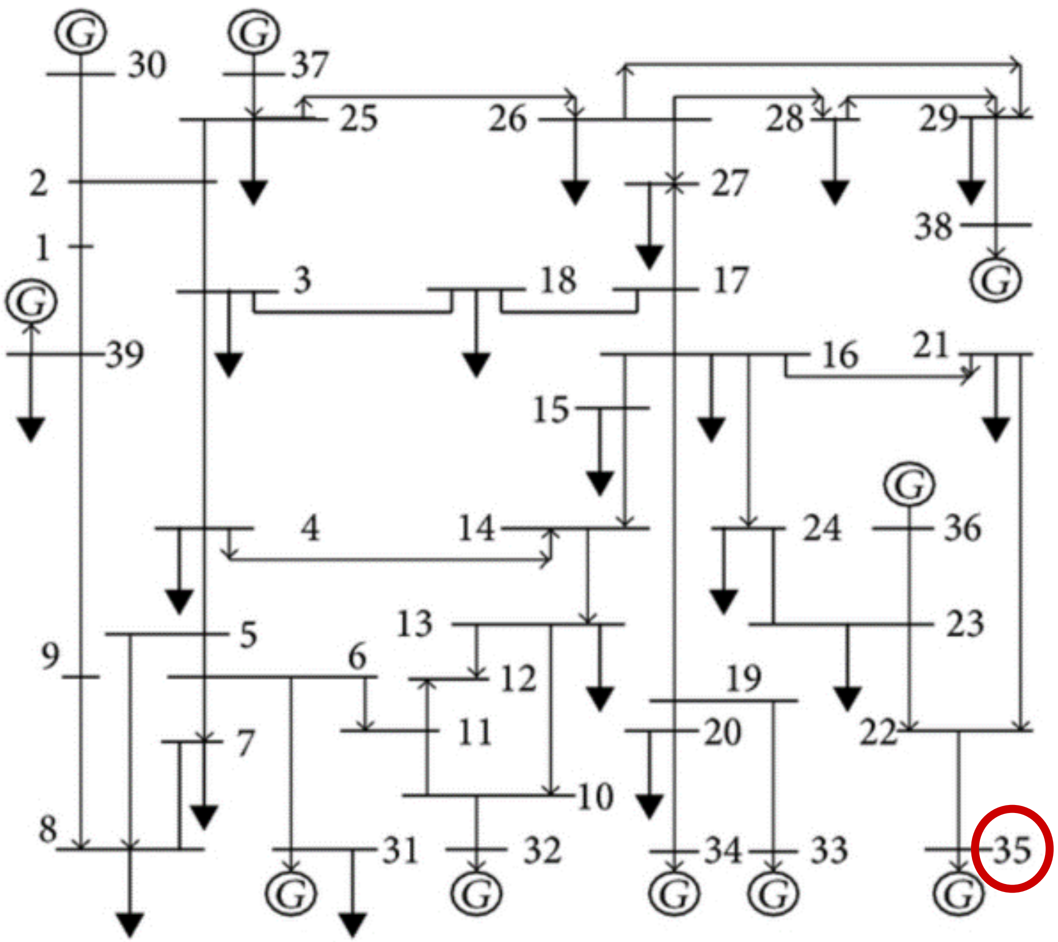


(a) {1,2,3,4,5,6}

(b) {1,18,13,8,29,33, 38}

Example 2 – Control Coverage

- Which choice provides controllability?



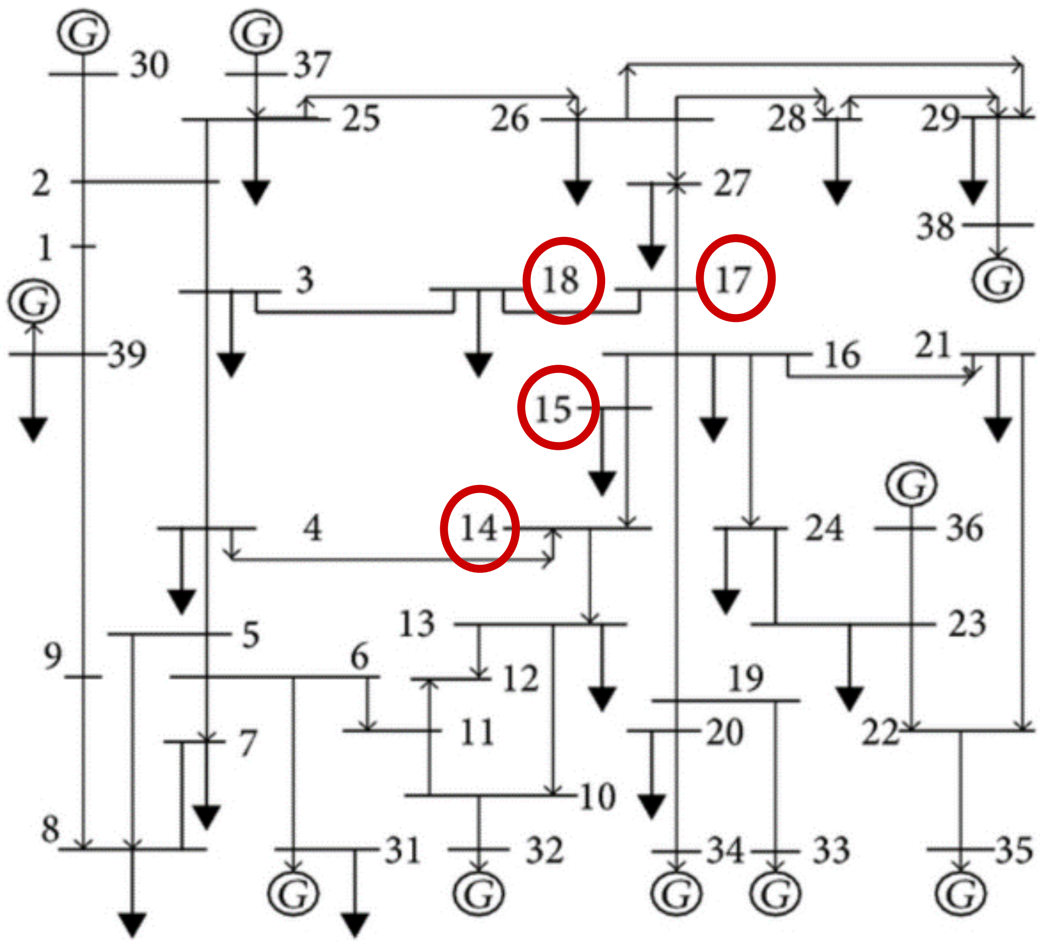
(a) {1,2,3,4,5,6}

(b) {1,18,13,8,29,33, 38}

(c) {35}

Example 2 – Control Coverage

- Which choice provides controllability?



(a) {1,2,3,4,5,6}

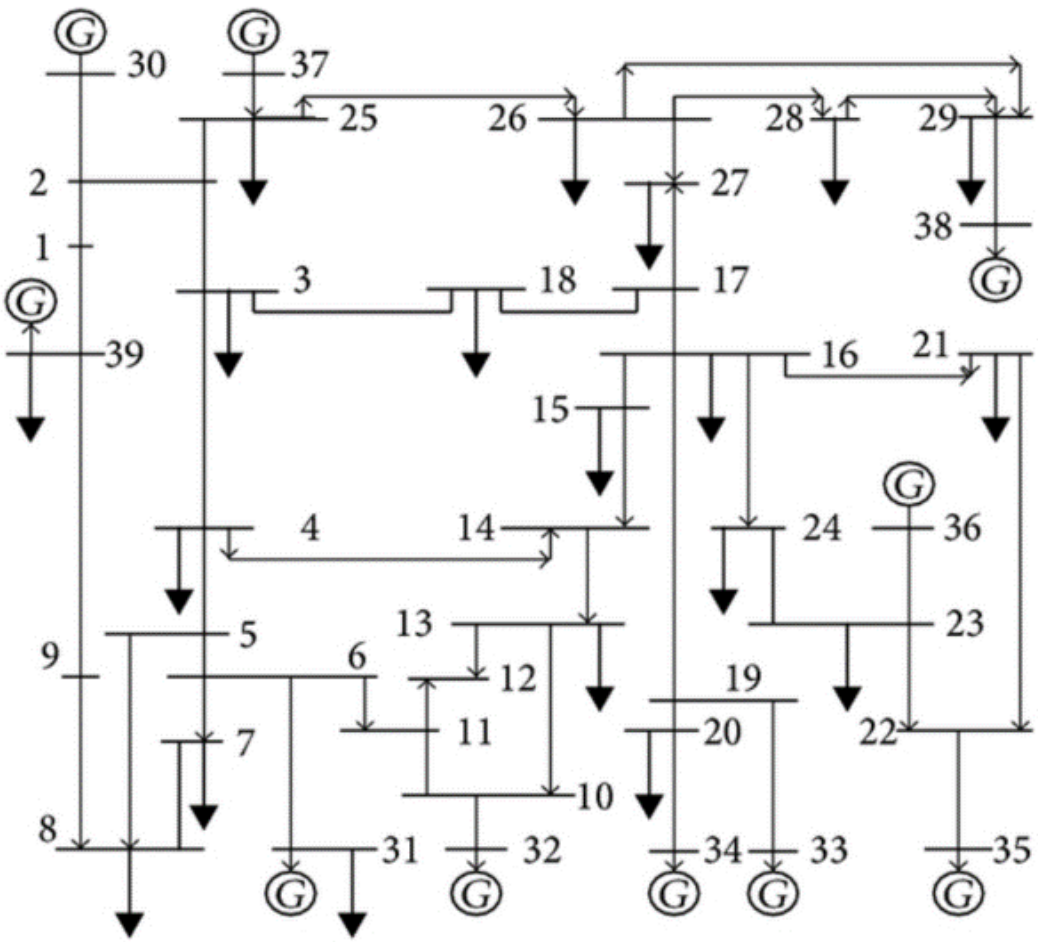
(b) {1,18,13,8,29,33, 38}

(c) {35}

(d) {14,15,17,18}

Example 2 – Control Coverage

- Which choice provides controllability?



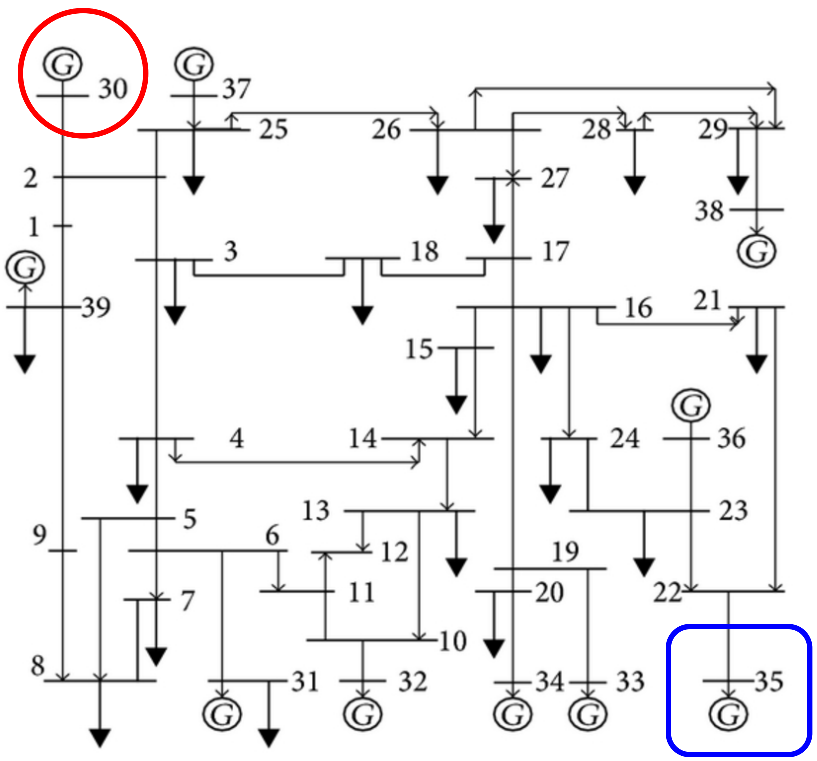
(a) {1,2,3,4,5,6}

(b) {1,18,13,8,29,33, 38}

(c) {35}

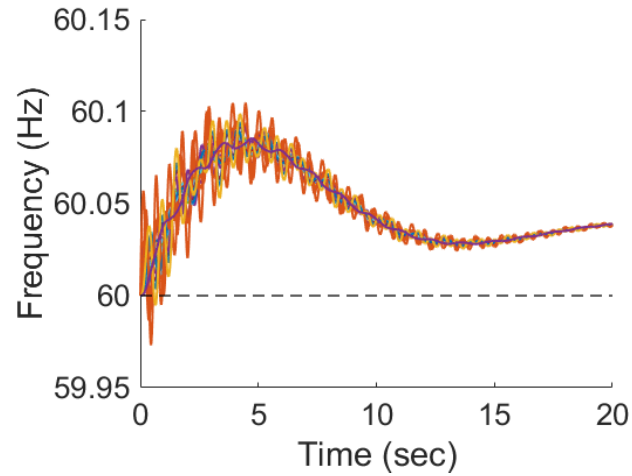
(d) {14,15,17,18}

Application

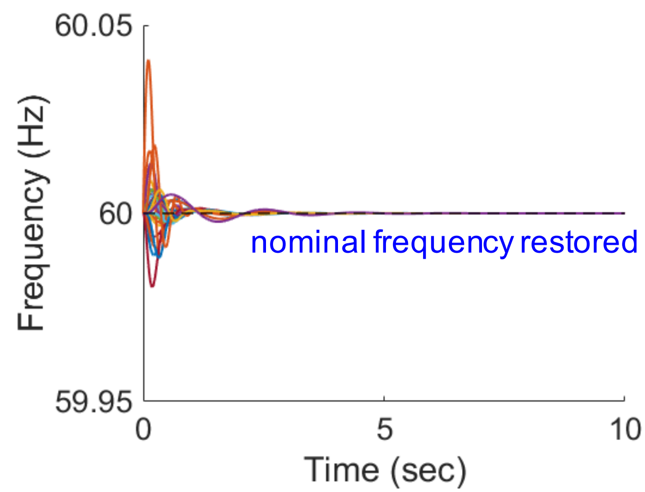


IEEE 39-bus New England system

1pu step disturbance at bus 30



without control



with local control at single bus 35

Summary

Relaxations of AC OPF

- Dealing with nonconvexity

Realtime AC OPF

- Dealing with volatility

Optimal placement

- Dealing with limited sensing/control