Duration-deadline Jointly Differentiated Energy Services

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Demand/supply balance with high renewables

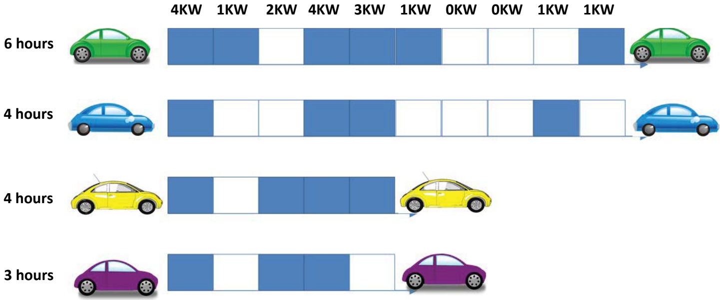
- Renewables bring in uncertainties and interruptions in the power supply.
- Conventional scheme of supply following demand may not work well.
  - Reserve generation is expensive.
  - Fast ramping requirement.
  - Create extra green-house gases.

Unlocking the power of load flexibilities

- An alternative scheme of demand following supply:
  Use the load flexibilities to compensate the supply uncertainties.
- Flexible loads can be modulated, deferred, or intermitted.
  - Thermostatically controlled loads (TCLs)
  - Electrical vehicles charging
  - Pool pumps


Duration-deadline jointly differentiated energy services

- Power delivery is segmented into a series of time slots.
- A load has both a duration requirement and a deadline requirement.
- A load is indifferent of the actual delivery time.
- Example: electrical vehicle charging.



Mathematical formulation

- T time slots. Power available at time slot j is c_j.
- N electric vehicles. EV i needs to be charged 1 KW for r_i time slots before deadline λ_i.
- The supply is adequate if there is a power allocation to meet all the demands.
- If further, $r_1 + r_2 + \cdots + r_N = c_1 + c_2 + \cdots + c_T$, supply is exact adequate.

Objectives

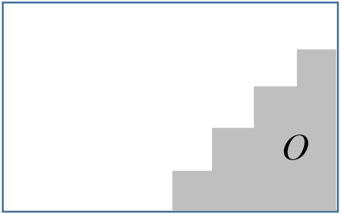
- Adequacy
  - What is the adequacy condition?
  - Given an inadequate supply, what is the minimum required purchase?
  - How to allocate? (not covered in this presentation)
- Market implementation
  - Social welfare problem
  - Existence of an efficient competitive equilibrium

Adequacy Problem

Adequacy: $(0,1)$-matrix completion problem

Complete a $(0,1)$-matrix A with a staircase of fixed zeros such that:

- the row sum vector is $r = \begin{bmatrix} r_1 & r_2 & \dots & r_N \end{bmatrix}'$
- the column sums are bounded by $c = \begin{bmatrix} c_1 & c_2 & \dots & c_T \end{bmatrix}'$



Significance of $(0, 1)$-matrix completion problems

- Practical significance:
  - Job allocation in data centers
  - Scheduling in real-time systems
  - Logistics
  - Image reconstruction
  - Graph realization
  - ...
- Theoretical significance:
  - Integer programming
  - Network flow theory
  - Matching theory
  - ...

Literature: unconstrained case

Gale-Ryser Theorem (1957)

The unconstrained $(0,1)$-matrix completion is solvable if and only if $c \prec^w r^*$.

- Conjugate vector r^*
  - Construct a $(0,1)$-matrix A^* with row sum vector r such that all the ones are put as far to the left as possible.
  - The column sum vector of A^*, is called the conjugate vector of r.
  - Example: $r = \begin{bmatrix} 2 & 3 & 4 & 6 \end{bmatrix}'$.

$$A^* = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$r^* = \begin{bmatrix} 4 & 4 & 3 & 2 & 1 & 1 \end{bmatrix}'.$$

Literature: unconstrained case

Gale-Ryser Theorem (1957)

The unconstrained $(0, 1)$-matrix completion is solvable if and only if $c \prec^w r^*$.

- Majorization:

$$x \prec y \text{ if } \sum_{i=1}^{k} x_i^{\uparrow} \geq \sum_{i=1}^{k} y_i^{\uparrow}, k = 1, \ldots, n-1, \text{ and } \sum_{i=1}^{n} x_i^{\uparrow} = \sum_{i=1}^{n} y_i^{\uparrow},$$

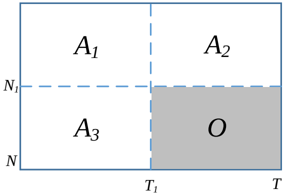
$$x \prec^w y \text{ if } \sum_{i=1}^{k} x_i^{\uparrow} \geq \sum_{i=1}^{k} y_i^{\uparrow}, k = 1, \ldots, n.$$

- A partial order that orders the level of fluctuations:
  - Example: $\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}' \preccurlyeq \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}'$.

Literature: constrained case

- Zero trace [Fulkerson, 1960]
- At most one fixed zero in each column [Anstee, 1982; Chen, 1992]
- Zero blocks on the diagonal [Lari, Ricca, and Scozzari, 2014]
- ...

Adequacy and adequacy gap (a two-deadline case)



- Define structure matrix S of dimension $T_1 \times (T - T_1)$:

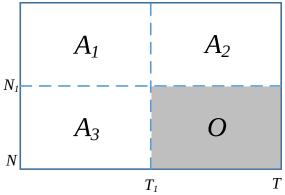
$$S_{k_1 k_2} = \sum_{j=k_1+1}^{T_1} c_j + \sum_{j=T_1+k_2+1}^{T} c_j - \sum_{i=1}^{N_1} [r_i - (k_1+k_2)]^+ - \sum_{i=N_1+1}^{N} (r_i - k_1)^+.$$

- The constrained $(0,1)$-matrix completion is solvable if and only if $S \geq 0$.
- In case of insufficient supply, the minimum additional power needed is

$$\left| \min_{k_1, k_2} S_{k_1 k_2} \right|.$$

W. Chen, Y. Mo, L. Qiu, and P. Varaiya, LAA, 2016

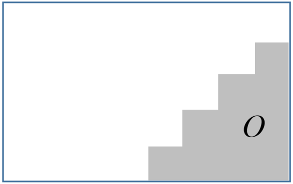
Interpretation



- Supply tail: $\sum_{j=k_1+1}^{T_1} c_j + \sum_{j=T_1+k_2+1}^{T} c_j$.
  Demand tail: $\sum_{i=1}^{N_1}[r_i - (k_1+k_2)]^+ + \sum_{i=N_1+1}^{N}(r_i - k_1)^+$.
- Energy dominance in tails.

$$S_{k_1 k_2} = \sum_{j=k_1+1}^{T_1} c_j + \sum_{j=T_1+k_2+1}^{T} c_j - \sum_{i=1}^{N_1}[r_i - (k_1+k_2)]^+ - \sum_{i=N_1+1}^{N} (r_i - k_1)^+ \geq 0.$$

Multiple deadlines: from structure matrix to tensor



- Structure tensor S:

$$S_{k_1 k_2 \ldots k_m} = \sum_{j=k_1+1}^{T_1} c_j + \sum_{j=T_1+k_2+1}^{T_2} c_j + \cdots + \sum_{j=T_{m-1}+k_m+1}^{T} c_j$$

$$-\sum_{i=1}^{N_1} [r_i - (k_1 + \ldots + k_m)]^+ - \sum_{i=N_1+1}^{N_2} [r_i - (k_1 + \ldots + k_{m-1})]^+ - \ldots - \sum_{i=N_{m-1}+1}^{N} (r_i - k_1)^+ .$$

W. Chen, Y. Mo, L. Qiu, and P. Varaiya, LAA, 2016

Market Implementation

Market implementation

Three elements of a market:

- Services:
  Duration-differentiated energy services with two different deadlines: T_1, T.
  The service of duration r and deadline λ has a price π_r^λ.
- Consumers:
  A continuum of consumers indexed by $x \in [0, 1]$.
  Utility function $U(x, p(x), r(x), \lambda(x))$.
- Supplier:
  An aggregate supplier who has available for free a supply profile c.

Social welfare optimization

- Consumer welfare: utility minus purchase cost.
  Supplier welfare: revenue minus production cost.
- Social welfare: total utility of the consumers: $\int_0^1 U(x, p, r, \lambda)dx$.
- Find an allocation that maximizes social welfare under adequacy constraint.

Theorem

The social welfare optimization problem has a solution for any type of utility function $U(x, p, r, \lambda)$.

W. Chen, L. Qiu, and P. Varaiya, CDC, 2015

Competitive equilibrium

- Consumers maximize their own welfare: $\max_{p,r,\lambda} U(x,p,r,\lambda) - p\pi_r^\lambda$.
- Supplier chooses production level n_r^λ for service (r,λ) to maximize revenue.
- Market clears: the level of consumption and production matches.
- A competitive equilibrium is said to be efficient if the resulting allocation maximizes the social welfare.

Theorem

There exists a forward market with an efficient competitive equilibrium.

W. Chen, L. Qiu, and P. Varaiya, CDC, 2015

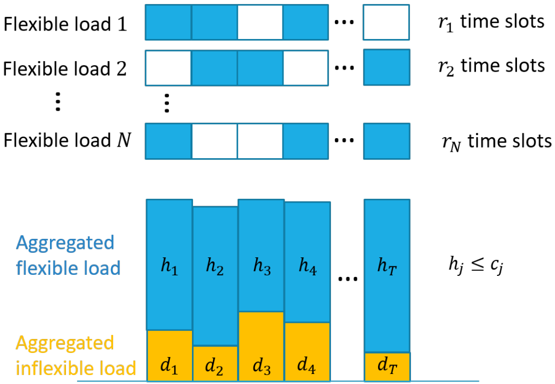
Summary

- Adequacy:
  - The adequacy condition is given by the nonnegativity of a structure tensor.
  - Adequacy gap: the largest difference between demand tails and supply tails.
- Market implementation:
  - Social welfare optimization has a solution for any type of utility functions.
  - The optimal social allocation can be sustained as a competitive equilibrium.

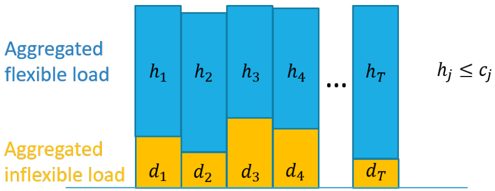
Extensions and beyond

- Multiple arrival and multiple deadlines.
  - Y. Mo, W. Chen, and L. Qiu, IFAC 2016
- Rate constrained energy services (an integer matrix completion problem).
  - Y. Mo, W. Chen, and L. Qiu, IFAC 2016
- Peer-to-peer charging (a $(-1, 0, 1)$-matrix completion problem).
  - Y. Mo, W. Chen, and L. Qiu, CDC 2016

Load balancing via optimization in majorization order



Load balancing via optimization in majorization order



- Optimization in majorization order:

$$\min_{\prec} \quad h + d$$
$$\text{subject to} \quad h \prec r^*, h \leq c.$$

- When c is sufficiently large, the minimum exists and can be achieved by a simple algorithm with complexity linear in NT.

Y. Mo, W. Chen, and L. Qiu, CDC, 2017